Turbulent mixed convection heat transfer to liquid sodium

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The influences of buoyancy on turbulent heat transfer to a liquid metal flowing in a vertical pipe are considered. A theoretical model is presented which provides a criterion for the conditions under which such influences become significant and which predicts the impairment of heat transfer for upward flow and enhancement for downward flow. The variation with Peclet number of the maximum impairment of heat transfer and conditions under which it occurs are established. A generalization of the model leading to an equation for the entire mixed convection region is proposed. From this an equation for turbulent free convection to liquid metals is obtained.

Key words: *heat transfer, liquid sodium, turbulence*

Over the past twenty years sodium-cooled fast breeder reactors have developed from an experimental stage to the point where they have become commercially feasible. Even so a recent review[1] has indicated that little information is available about mixed convection to liquid sodium, even for the simplest of situations such as flow in uniformly heated smooth vertical pipes. However, the problem has been studied extensively for other fluids such as water, carbon dioxide and air[2].

A theoretical model of turbulent mixed convection in vertical tubes has been proposed[3,4] to explain effects observed with supercritical pressure fluids. It has subsequently been developed and used successfully to correlate data for other fluids[5–7]. The model is extended here to cover the case of low Peclet number.

The effects of buoyancy on turbulent heat transfer

The system under consideration is a heated pipe with turbulent flow in the upward or downward direction. The fluid is a liquid metal such as sodium.

As the temperature difference between surface and fluid is increased the boundary layer near to the heated surface experiences a buoyancy force because of its reduced density and this changes the shear stress distribution within it (see Fig 1). A consequence of the modification to the shear stress is that turbulence production is affected, the flow becoming similar to that at reduced Reynolds number for upward flow and increased Reynolds number for downward flow. This leads to impairment and enhancement of heat transfer relative to that for the buoyancy-free condition.

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The change of shear stress across the buoyant layer is given by the integral:

$$\Delta\tau_{\delta_B} = \int_0^{\delta_B} (\rho_b - \rho) g\, dy \tag{1}$$

Liquid metals have a gradual variation of density with temperature so that the buoyant layer and thermal layer are effectively identical, ie $\delta_B = \delta_T$. Hence,

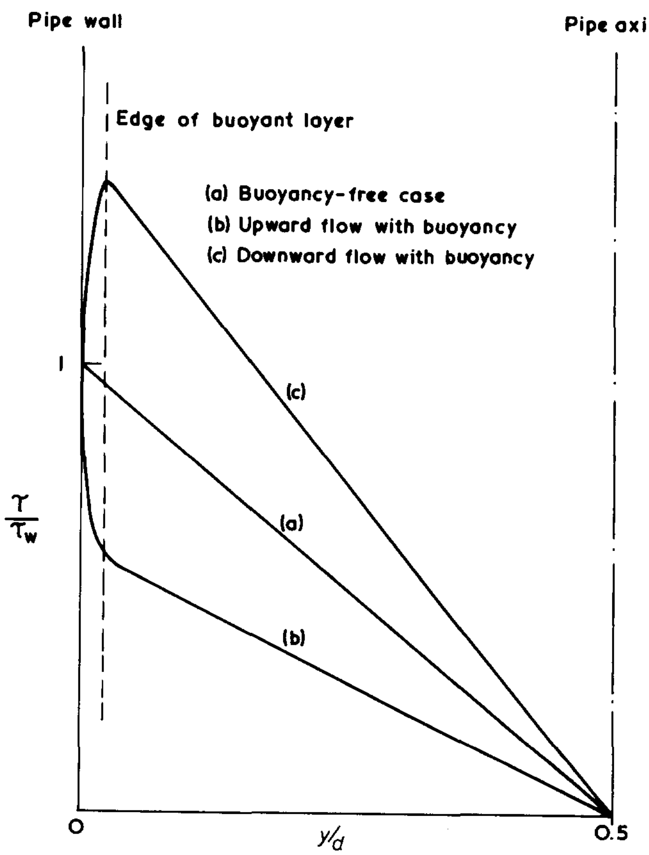

Fig. 1 Influence of buoyancy on distribution of shear stress

approximating the temperature gradient in the thermal layer by $(T_w - T_b)/\delta_T$ it is possible to replace Eq (1) by:

$$\Delta\tau_{\delta_B} = \frac{\delta_B g}{(T_w - T_b)} \int_{T_b}^{T_w} (\rho_b - \rho)\, dT \qquad (2)$$

As the variation of ρ with T can be described to a good accuracy by a linear relation this integral can be evaluated to give:

$$\Delta\tau_{\delta_B} = (\rho_b - \rho_w) g \delta_B / 2 \qquad (3)$$

Making use of the known dependence of heat transfer on Prandtl number for liquid metals δ_T can be related to δ_M, the thickness of the sublayer/buffer layer, by:

$$\delta_T = \delta_M / \mathrm{Pr}^{0.8} \qquad (4)$$

Noting that δ_T can be replaced by δ_B the following equation for $\Delta\tau_{\delta_B}$ is obtained:

$$\Delta\tau_{\delta_B} = \tfrac{1}{2}(\rho_b - \rho_w) g \delta_M / \mathrm{Pr}^{0.8} \qquad (5)$$

A dimensionless wall-layer thickness δ_M^+ defined by:

$$\delta_M^+ = (\tau_w \rho)^{1/2} \delta_M / \mu$$

is introduced next. In line with established ideas a value of 20 is assigned to δ_M^+ and thus the fractional change in shear stress due to buoyancy, $\Delta\tau_{\delta_B}/\tau_w$, becomes:

$$\frac{\Delta\tau_{\delta_B}}{\tau_w} = \frac{10\mu(\rho_b - \rho_w) g}{\tau_w^{3/2} \rho^{1/2} Pr^{0.8}} \qquad (6)$$

Introducing the dimensionless parameters $Gr = (\rho_b - \rho_w) g d^3 / \rho \nu^2$, $Re = \rho u_b d / \mu$ and $C_f = \tau_w / \frac{1}{2}\rho u_b^2$ and utilizing the empirical relation $C_f = 0.046\, Re^{-0.2}$, Eq (6) becomes:

$$\frac{\Delta\tau_{\delta_B}}{\tau_w} = \frac{3000\, Gr}{Re^{2.7} Pr^{0.8}} \qquad (7)$$

Likening the buoyancy-influenced situation to that of a buoyancy-free one at a modified Reynolds number, Re', with wall shear stress τ'_w, the modified and nominal values of Reynolds number and shear stress can be related by:

$$\frac{Re'}{Re} = \left(\frac{\tau'_w}{\tau_w}\right)^{1/1.8} \qquad (8)$$

Using the established empirical form of equation relating Nu to Re for low Peclet number forced convection, ie $Nu = 5 + 0.025\, Re^{0.8} Pr^{0.8}$, the modified Nusselt number can be expressed in terms of Nu, τ_w and τ'_w as:

$$Nu' = 5 + (Nu - 5)\left(\frac{\tau'_w}{\tau_w}\right)^{0.8/1.8} \qquad (9)$$

The modified stress, τ'_w, is then related to $\Delta\tau_{\delta_B}$ by:

$$\tau'_w = \tau_w \pm \Delta\tau_{\delta_B} \qquad (10)$$

where the positive sign applies for the downward flow case and the negative sign for upward flow. Combining Eqs (7), (9) and (10) and associating the modified Nusselt number, Nu', with that for buoyancy-influenced heat transfer, Nu_B:

$$Nu_B = 5 + 0.025\, Re^{0.8}\, Pr^{0.8}\left(1 \pm \frac{3000\, Gr}{Re^{2.7}\, Pr^{0.8}}\right)^{0.445} \qquad (11)$$

Hence, the ratio of Nusselt number for mixed convection to Nusselt number for forced convection, expressed in terms of Grashof number, Reynolds number and Peclet number, becomes:

$$\frac{Nu_B}{Nu_F} = \frac{5 + 0.025\, Pe^{0.8}\left(1 \pm \frac{3000\, Gr}{Re^{1.9} Pe^{0.8}}\right)^{0.445}}{5 + 0.025\, Pe^{0.8}} \qquad (12)$$

The effects of buoyancy predicted by this equation are shown in Fig 2. For the upward flow case the curves terminate at the point where the

Notation

C_p	Specific heat at constant pressure (kJ/kg)
C_f	Friction coefficient, $C_f = \tau_w / \frac{1}{2}\rho u_b^2$
d	Tube diameter (m)
L	Tube length (m)
g	Acceleration due to gravity (m/s^2)
k	Thermal conductivity (kW/mK)
q_w	Heat flux at the wall (kJ/m^{2}s)
T	Temperature (°C or K)
T_b	Bulk temperature (°C or K)
T_w	Wall temperature (°C or K)
u_b	Bulk velocity (m/s)
y	Transverse coordinate measured from wall (m)
α	Heat transfer coefficient, $\alpha = q_w / (T_w - T_b)$(kW/m^2 K)
δ_B	Thickness of buoyant layer (m)
δ_M	Thickness of sub-layer plus buffer layer (m)
δ_T	Thickness of thermal layer (m)
μ	Dynamic viscosity (kg/ms)
ρ	Density (kg/m^3)
ρ_b	Density evaluated at bulk temperature (kg/m^3)
ρ_w	Density evaluated at wall temperature (kg/m^3)
ν	Kinematic viscosity, $\nu = \mu/\rho$ (m^2/s)
τ	Local value of total shear stress (N/m^2)
τ_w	Wall shear stress (N/m^2)
$\Delta\tau_{\delta_B}$	Change of shear stress across buoyant layer
Re	Reynolds number, $Re = \rho u_b d/\mu$
Pr	Prandtl number, $Pr = \mu C_p / k$
Pe	Peclet number, $Pe = \rho u_b C_p d / k$
Nu	Nusselt number, $Nu = \alpha d / k$
Nu_F	Nusselt number for forced convection
Nu_B	Nusselt number for buoyancy-influenced (mixed) convection
Gr	Grashof number, $Gr = (\rho_b = \rho_w) d^3 g / \rho \nu^2$
δ_M^+	Dimensionless wall-layer thickness, $\delta_M^+ = \delta_M (\tau_w \rho)^{1/2} / \mu$

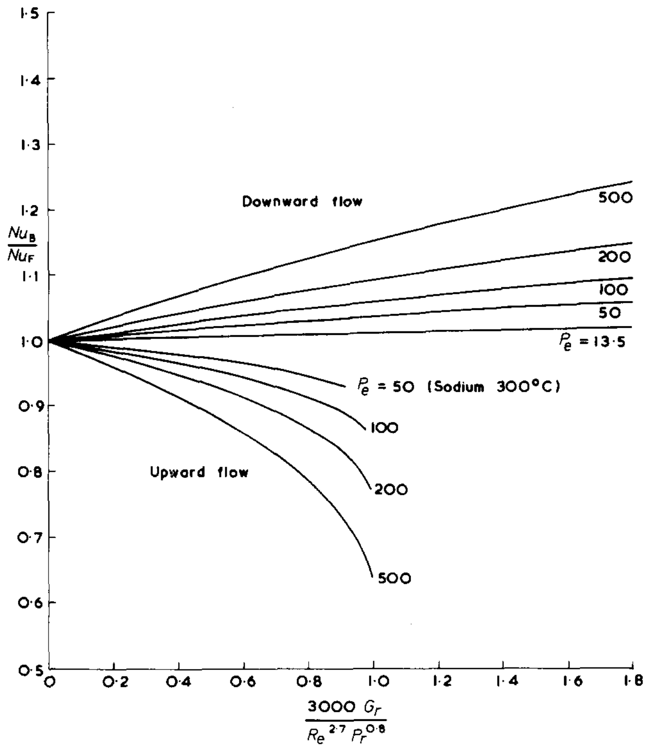

Fig 2 Predicted effects of buoyancy

reduced value of Reynolds number attains the critical value for reversed transition, Re_{crit}, (laminarization of the flow). Combining Eqs (7), (8) and (10) the corresponding critical value of the buoyancy parameter $3000\,Gr/Re^{1.9}\,Pe^{0.8}$ is:

$$\left(\frac{3000\,Gr}{Re^{1.9}Pe^{0.8}}\right)_{crit}=1-\left(\frac{Re_{crit}}{Re}\right)^{1.8} \tag{13}$$

The Nusselt number ratio at the end points is:

$$\left(\frac{Nu_B}{Nu_F}\right)_{crit}=\frac{5+0.025\,Pe_{crit}^{0.8}}{5+0.025\,Pe^{0.8}} \tag{14}$$

Hence, the percentage impairment of heat transfer at the laminarized condition is given by:

$$\frac{(Pe^{0.8}-Pe_{crit}^{0.8})}{(Pe^{0.8}+200)}\times 100 \tag{15}$$

Fig 3 shows curves obtained using this expression for sodium at temperatures in the range 150–600 °C.

For values of buoyancy parameter in excess of critical, heat transfer would be expected to be better than that at laminarization. However, the theoretical model does not allow predictions to be made for laminar conditions. Further increase in the buoyancy parameter will lead eventually to buoyancy-induced turbulent transition and another regime of turbulent mixed convection (see later).

Criterion for the onset of buoyancy effects

It is of some practical importance to be able to determine the conditions under which buoyancy effects begin to be significant in liquid metal heat transfer. A criterion can be arrived at by considering the limiting form of Eq (12) for the case where the

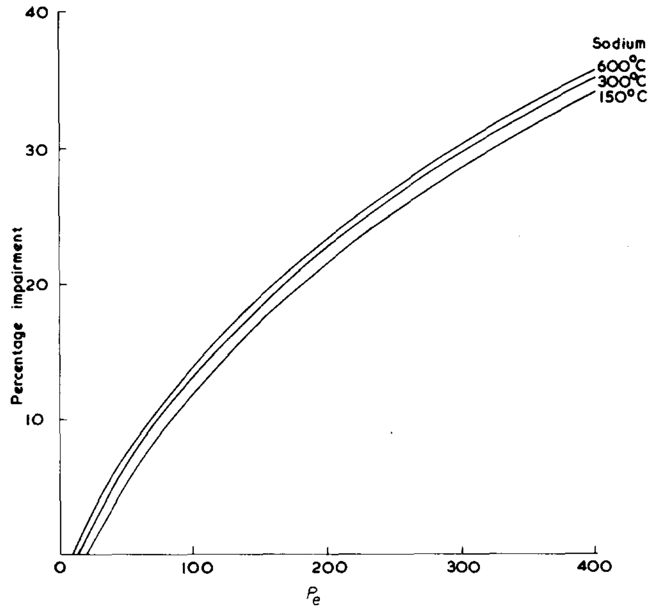

Fig 3 Maximum impairment of heat transfer

buoyancy parameter is small. For such conditions Eq (12) reduces to:

$$\frac{Nu_B}{Nu_F}=1\pm\frac{6.675\,Gr}{(1+0.005\,Pe^{0.8})\,Re^{1.9}} \tag{16}$$

Thus, for influences of buoyancy to be greater than about 2%

$$\frac{Gr}{(1+0.005Pe^{0.8})Re^{1.9}}>3\times10^{-3} \tag{17}$$

For the range of Peclet number of interest in sodium work this effectively reduces to:

$$\frac{Gr}{Re^2}>2\times10^{-3} \tag{18}$$

and in this form it can be compared directly with an empirical criterion[8] which appears to be the only one currently available for practical purposes[1]. This takes the form:

$$4\frac{d}{L}\frac{Gr\,Nu}{Re^2}>2\times10^{-3} \tag{19}$$

Noting that the product $4Nu(d/L)$ will in practice be of order unity, the criterion yielded by the present model is seen to be in good agreement with the empirical one.

In Fig 4 the predictions of the present theory are plotted against the parameter $Gr/(1+0.005\,Pe^{0.8})Re^{1.9}$. Also shown are curves given by the equation:

$$\frac{Nu_B}{Nu_F}=\left[1\pm\frac{15\,Gr}{(1+0.005\,Pe^{0.8})Re^{1.9}}\right]^{0.445} \tag{20}$$

which is an extended version of Eq (16) taking some account of higher order terms. It can be seen that whereas a reasonable correlation of the curves for different Peclet number is obtained for downward flow the results for upward flow show significant spread and are not well represented by Eq (20).

Generalized equation for mixed convection

This theoretical model presented utilizes empirical relations for friction and heat transfer based upon data for buoyancy-free conditions. In that respect it is essentially an extrapolation into the mixed convection region from the forced convection end. The model would not be expected to be applicable to conditions where buoyancy effects are dominant, ie at the free convection end, and certainly Eq (12) does not satisfy the requirement that Nu_B should become independent of Re for such conditions. However, using an approach which was developed in an earlier paper[5], concerning mixed convection heat transfer to water with downward flow, Eq (12) can be made to do so by changing the index 0.445 to the value 0.3. If in addition the constant 3000 is changed to 5000 the resulting equation gives values of Nu_B/Nu_F which are virtually identical with those from Eq (12) over the range for which the theoretical model could reasonably be expected to be applicable. Fig 5 shows comparisons between the theory for downward flow (Eq (12)) and the generalized equation:

$$\frac{Nu_B}{Nu_F}=\frac{5+0.025\,Pe^{0.8}\left(1+\dfrac{5000\,Gr}{Re^{1.9}Pe^{0.8}}\right)^{0.3}}{5+0.025\,Pe^{0.8}} \qquad (21)$$

Also shown are the theoretical curves for upward flow and curves given by the following modification of Eq (21) for upward flow with strong influences of buoyancy and buoyancy-induced turbulence production:

$$\frac{Nu_B}{Nu_F}=\frac{5+0.025\,Pe^{0.8}\left(\dfrac{5000\,Gr}{Re^{1.9}Pe^{0.8}}-1\right)^{0.3}}{5+0.025\,Pe^{0.8}} \qquad (22)$$

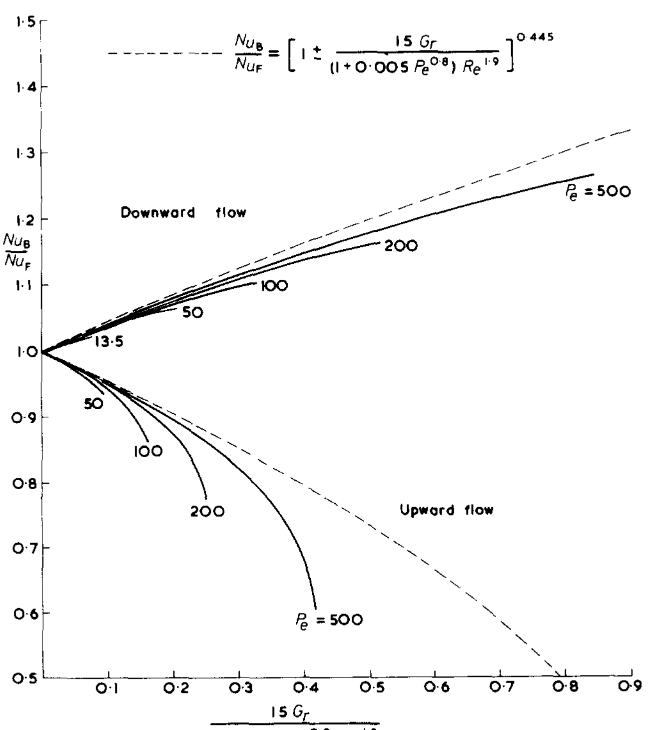

Fig 4 Comparison with limiting equation

These curves together provide a general picture of mixed convection heat transfer to liquid metals which is similar to that exhibited by data for other fluids[2].

Turbulent free convection

In the limit where buoyancy-induced flow completely dominates the forced flow the upward and downward flow cases become identical and the process is effectively one of turbulent free convection. Under such conditions Eqs (21) and (22) both reduce to:

$$Nu_B=5+0.4\,(Gr\,Pr^2)^{0.3} \qquad (23)$$

Some experimental data are available[7] for liquid sodium flowing in a passage of rectangular cross-section under conditions of mixed convection with

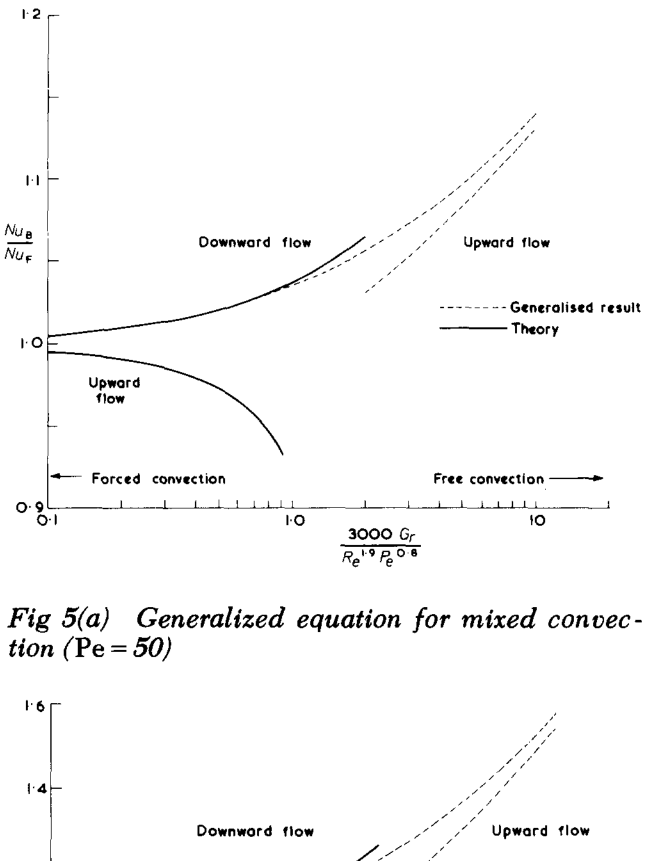

Fig 5(a) Generalized equation for mixed convection (Pe = 50)

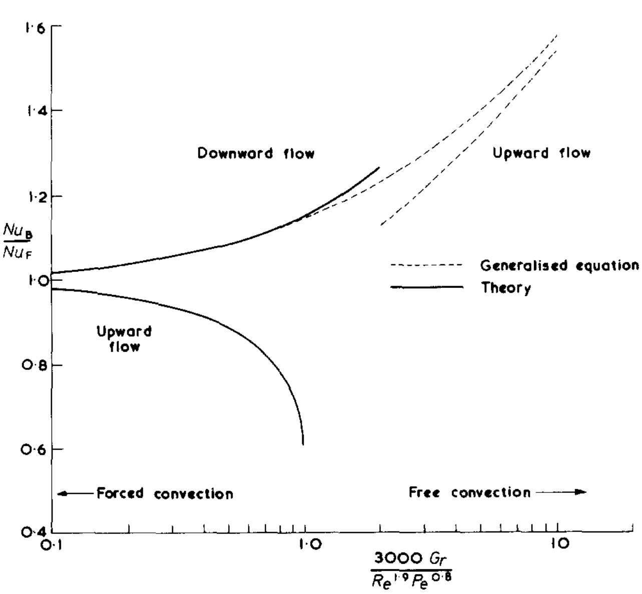

Fig 5(b) Generalized equation for mixed convection (Pe = 500) very strong influences of buoyancy. However, direct comparison of the data with Eq (23) is difficult because of the form in which the results are published. The best that can be said is that the Nusselt numbers quoted for conditions of maximum buoyancy effect are similar to those indicated by Eq (23). Data on free convection to liquid metals appears to be limited to laminar conditions. Comparison of Eq (23) with such results obtained using mercury[10] indicates, as expected, that it significantly overestimates the Nusselt number. Exact theoretical predictions are possible for the laminar flow case and Fig 6 shows a comparison between Eq (23) and such predictions in the region of turbulent transition. It can be seen that whereas Eq (23) gives values of about 24 the laminar flow value is 17, a surprisingly close and plausible result.

Conclusions

Buoyancy forces cause enhancement of turbulent heat transfer to liquid metals for downward flow and impairment for upward flow. For $Gr/Re^{1.9}\,Pe^{0.8} < 3\times10^{-4}$ the effects are described by the equation:

$$\frac{Nu_B}{Nu_F}=\frac{5+0.025\,Pe^{0.8}\left(1\pm\frac{3000\,Gr}{Re^{1.9}\,Pe^{0.8}}\right)^{0.445}}{5+0.025\,Pe^{0.8}}$$

where the positive sign applies to downward flow and the negative sign to upward flow. These effects of buoyancy on heat transfer to liquid metals are less marked than for other fluids because of the relatively reduced influence of turbulent diffusion at low Peclet number.

A criterion for onset of significant buoyancy effects in liquid sodium is:

$$\frac{Gr}{Re^2}>2\times10^{-3}$$

Maximum impairment of heat transfer in the upward flow case is associated with laminarization

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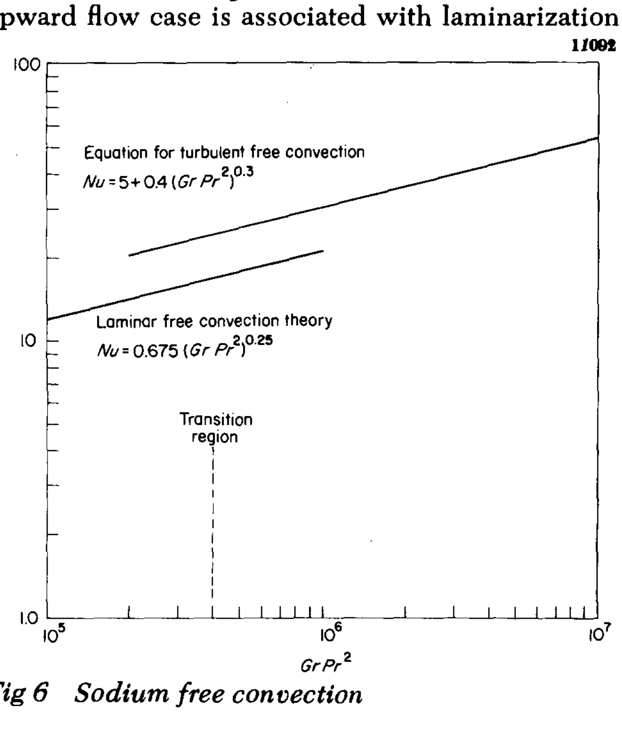

Fig 6 Sodium free convection

of the flow and increases with Peclet number. It occurs when:

$$\frac{Gr}{Re^{1.9}Pe^{0.8}}\sim3\times10^{-4}$$

Expressed as a percentage of the heat transfer coefficient for forced convection, the predicted maximum impairment for liquid sodium is given by:

$$\frac{(Pe^{0.8}-10)}{(Pe^{0.8}+200)}\times100$$

For downward flow a generalized equation describing mixed convection over the entire range of possible conditions from forced to free convection has the form:

$$\frac{Nu_B}{Nu_F}=\frac{5+0.025\,Pe^{0.8}\left(1+\frac{5000\,Gr}{Re^{1.9}Pe^{0.8}}\right)^{0.3}}{5+0.025\,Pe^{0.8}}$$

For conditions of strong influence of buoyancy, enhancement of heat transfer occurs for both upward and downward flow, the mechanism being one of buoyancy-induced turbulence production. In the limiting condition where the process is effectively one of turbulent free convection the two cases coincide and the equation for heat transfer which is arrived at from the present considerations is:

$$Nu=5+0.4(Gr\,Pr^2)^{0.3}$$

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